AN INVESTIGATION OF THE VALUE OF MINIMIZING OBJECTIVE FUNCTIONS OVER THE EFFICIENT SET IN MULTIPLE CRITERIA DECISION MAKING WITH AN APPLICATION TO THE CITRUS ROOTSTOCK SELECTION PROBLEM IN FLORIDA

BY

DONGYEUP LEE

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In this dissertation, a multiple objective linear programming model for the citrus rootstock selection problem in Florida is developed. Its application to a real-world problem in the selection of Florida citrus rootstocks is also presented. The results demonstrate that the model developed in this dissertation could be used at any citrus enterprise in Florida if the necessary data were available.

The model for the rootstock selection problem in Florida is solved by two different methods, each using the interactive STEM procedure. The first method uses solutions obtained by a payoff table approach. The second method uses solutions obtained by the Benson-Sayin heuristic approach. Before using the Benson-Sayin method, we conducted computational experiments to test its suitability for our problem of finding

the estimate of a minimum criterion value over the efficient set. We found that the Benson-Seyin method can be used quite practically and efficiently with our problem. The results using the two approaches to solve the Florida citrus rootstock selection model are compared and analyzed in order to detect possible differences in the efficiency of the STEM method when implemented in these two different ways. These comparisons show that the efficiency of the STEM procedure can increase by using the solutions obtained from the Benson-Seyin heuristic method instead of using the solutions obtained from payoff table method.

CHAPTER 1

INTRODUCTION

Decision making is a process by which an alternative is selected as being "preferred" from among a set of alternatives. The decision process involves a model, or a logical structure, which is a simplified representation of reality. This model or structure enables the decision maker (DM) to impose some sort of preference upon the variables involved in the model. The DM requires information concerning the feasibility of alternative courses of action and the means to evaluate their respective results. The termination of the process, which yields a decision, is the selection of a particular alternative.

Often the selection of an alternative is the result of a comparision of the worth of feasible alternatives. In this case the choice is simply the selection of the alternative with the greatest worth. It is, however, important to notice that the comparision and resulting decision reflect the value structure of the individual decision maker at the time of the choice. This value structure can vary from individual to individual and for the same individual from time to time.

Optimization theory has been introduced to the decision making process with regard to both the development and selection of alternatives. Multicriteria problems have been of increasing interest to management scientists, due in part to the realization that many problems, particularly those of a strategic nature, and more particularly those in the public sector, must explicitly consider multiple criteria if they are to be resolved with truly good decisions. For example, in private business, while profit maximization is still a very important objective, today's business environment requires that the business manager seek the attainment of other objectives, such as good will, energy conservation, environmental protection, improved labor relations, observance of social responsibilities, and attention to government regulations, sometimes with even higher priorties on the noneconomic objectives.

In this dissertation, a multiple criteria decision making model is developed in order to search for the best compromise solution to the citrus rootstock selection problem in Florida.

We believe that the multiple criteria decision making model developed in this dissertation could help citrus growers in Florida make a sound rootstock selection decision.

Recently, some progress has been made in the theory and practice of multiple criteria decision making. A number of different techniques are presently available to help decision makers handle multiple criteria decision making problems such

as the citrus rootstock selection problem in Florida. In practice, interactive methods have proven to be most effective in generating 'good' compromise solutions for multiple criteria decision making problems (Steuer, 1986, p.4 and p.361).

In the multiple criteria decision making problem, minimum criterion values over the efficient set (defined in Section 1.2) are of interest in order to characterize the ranges of the criterion values over the efficient set. In fact, a number of interactive methods (Benayoun, De Montgolfier, Tergny and Laritchev, 1971; Belenson and Kapur, 1973; Spronk and Telgen, 1981; Kok and Lootsma, 1985) utilize payoff tables (defined in Section 3.1) to obtain estimates of the minimum criterion values over the efficient set. Isermann and Steuer (1987) pointed out that these interactive methods could benefit if better methods than the method using payoff tables were used in order to find or estimate minimum criterion values over the efficient set. Relatively few attempts (Dessouky, Ghiassi and Davis, 1986; Isermann and Steuer, 1987) have been made to find or estimate minimum criterion values over the efficient set. In addition to these attempts, Philip (1972) and Benson (1984, 1986, 1991) have considered the problem of finding minimum criterion values over the efficient set as a special case of their problem, which is the problem of optimizing a linear function over the efficient set.

In this dissertation, the improvement of the efficiency of one of these interactive methods in the process of solving the citrus rootstock selection problem will be investigated by using a heuristic method recently developed by Benson and Sayin (1991).

The general forms of the multiple criteria decision making problem (MCDM) and the multiple objective linear programing problem (MOLP) are introduced in Section 1.1. In Section 1.2, the problem of optimizing a linear function over the efficient set will be discussed. In Section 1.3, a discussion of citrus rootstock use in the citrus industry will be presented. This will be followed in Section 1.4 by an outline of the content of this dissertation. The notation and definitions which are presented will be used throughout the remainder of the dissertation.

1.1 An Overview of The Multiple Criteria Decision Making Problem (MCDM)

The most important reason for increasing interest in the MCDM problem is the recognition that most decision problems are inherently multiobjective. Even many problems addressed by classical single-objective models can easily be viewed as multiobjective in nature. Examples of such problems include project management problems (Talbot 1982), inventory planning problems (Kendall and Lee 1980), scheduling problems (Huckert et al. 1980) and capacity expansion problems (Evans et al.

1982). Another important reason for increasing interest in the MCDM problem is the enormous improvement over the past 20 years in the speed, storage, and flexibility of computing facilities. Algorithms for solving MCDM problems typically require much more storage and CPU time than algorithms that address similar single objective models. In addition, many of the multiobjective algorithms require an interactive approach (see section 2.2) between the decision maker and the computer. These interactive approaches necessitate speedy responses from the computer and flexibility in computing hardware and software.

The term, multiple criteria decision making problem, refers to a decision problem with two or more objective (criterion) functions. The multiple criteria decision making problem differs from the single objective optimization problem only in the expression of the respective objective functions.

The single criterion decision making problem (SCDM) can be written as

$$\begin{aligned} &\text{Max} \quad f(x) \qquad\qquad \text{(SCDM)} \\ &\text{subject to} \quad x \in X, \end{aligned}$$

where x is an n-dimensional vector of decision variables, the objective function f(x) is a scalar function of the vector x, and X represents the constraint set.

The multiple criteria decision making problem (MCDM) can, in general, be written

$$\text{Max} \quad f(x) = (f_1(x),\ f_2(x), \ldots,\ f_p(x)) \quad \text{(MCDM)}$$
$$\text{subject to} \quad x \in X,$$

where x and X are defined as in the single criterion decision making problem and f(x) is a p-dimensional vector of objective functions.

For convenience, in the remainder of this dissertation, for any two vectors x and y of the same dimension, $x \geq y$ will denote that $x \geq y$ and $x \neq y$. Also, the vector of objective functions will be denoted $f(x)=(f_1(x),f_2(x),\ldots,f_p(x))$.

In the multiple criteria decision making (MCDM) problem, 'maximization' is not well defined since the objective functions may be conflicting with each other, and usually some compromise solution is required. Numerous techniques to find the most preferred compromise solution have been proposed in the literature, where 'most preferred' depends upon the preferences of the decision maker (DM). Usually the most preferred compromise solution is required to be an efficient (nondominated, Pareto) or weakly efficient solution.

Definition 1.1 A point $x^0 \in X$ is an efficient solution of problem (MCDM) if and only if there exists no $x \in X$ such that $f(x) \geq f(x^0)$.

Definition 1.2 A point $x^0 \in X$ is a weakly efficient solution of problem (MCDM) if and only if there exists no $x \in X$ such that $f(x) > f(x^0)$.

The origin of the concept of efficiency is in the work of Pareto (1896). Later, Kuhn and Tucker (1951) introduced the notion of using a vector-valued objective function in mathematical programming and derived necessary and sufficient conditions for a solution to be efficient. Unfortunately, there were only scattered discussions concerning multiple objectives during the fifties, possibly due to extensive interest and development in single-objective optimization. Much of the research in MCDM has occurred in the last twenty years (Benson, 1978,1979, Ecker and Kauada, 1978; Evans and Steuer, 1973; Geoffrion, 1968; Soland, 1979; Steuer, 1986; Yu, 1974; Yu and Zeleny, 1975; Zeleny, 1982).

1.1.1 The Multiple Objective Linear Programming Problem (MOLP)

One of the more popular and practical models that has been used to help make decisions involving multiple criteria is the multiple objective linear programming problem (MOLP) model.

This model can be written

$$\begin{array}{lll} \text{Max} & Cx & \qquad\qquad\qquad \text{(MOLP)} \\ \text{subject to} & x \in X, & \end{array}$$

where C is a p×n matrix whose rows C_i, $i=1,2,\ldots,p$, are the coefficients of the p linear criterion functions, and $X \subseteq \mathbb{R}^n$ is a polyhedron.

From Definition 1.1, a point $x^0 \in X$ is said to be an efficient solution for problem (MOLP) when there is no $x \in X$ such that $Cx \geq Cx^0$. This means that a solution is efficient if it is not possible to improve the achievement of any single linear objective function without worsening the achievement of at least one other linear objective function.

1.1.2 Solution Methods for the MOLP Problem

Solution procedures for the MOLP problem differ depending on how much preference information is requested from the decision maker (DM) and when it is requested in the decision making process (either before, during, or after problem solution).

At one extreme, if complete and accurate preference information is available from the DM prior to problem solution, then the MOLP problem can be reduced to a single objective optimization problem and solved for an optimal

solution. However, it is usually unrealistic to expect a DM to be able to supply complete and accurate preference information prior to the solution of a MOLP problem. In the absence of this preference information, there is usually no single optimal solution to a MOLP problem (a solution which maximizes all of the objective functions simultaneously). Therefore, the concept of optimality is often replaced by that of efficiency. It is argued that the DM's most preferred compromise solution to a MOLP problem should be efficient. Otherwise, as long as the DM prefers "more rather than less," there would exist at least one other feasible solution which the DM would prefer to the current compromise solution.

At the other extreme, if no preference information is requested from the DM prior to solution, the analyst could try to generate all of the efficient or efficient extreme point solutions to the MOLP problem and present them to the DM. This approach has been applied to MOLP problems by a number of authors, including Evans and Steuer (1973), Yu and Zeleny (1975), Ecker and Kouada (1978), and Steuer (1986). However, the identification of the efficient set through enumeration of its efficient or efficient extreme points is usually a computationally difficult task, since the number of efficient extreme point solutions can be quite large. Furthermore, even though the identification of the efficient set may be useful to the decision maker in narrowing down the range of the search, it may not be helpful enough to encourage

him to utilize this approach for the following two reasons. First, the size of the efficient set may be too large and its shape too complex to visualize. Second, no guidelines are provided to help the DM to understand the relationship among points in the set of efficient solutions and the tradeoffs that they represent to his internal preference structure.

In between these two extremes are interactive procedures. These procedures generate subsets of solutions for the DM to process with the aid of a computer. During an interactive procedure, the DM is required to provide some information concerning his preferences over the generated solutions. In this way, interactive procedures allow the DM's preferences to evolve over time as he gains more knowledge of the problem and its solutions. The procedure continues until the DM or the computer program identifies the current solution as a most preferred solution or a best compromise solution. Some of the major advantages of interactive methods are as follows:

1. There is no need for a priori preference information which is quite difficult for the DM to provide.

2. Interactive methods provide a learning process for the DM to understand the behavior of the problem.

3. The DM can learn about his preferences which are often initially vague and not exactly known.

Even though the solutions obtained in interactive methods depend upon the accuracy of the information that the DM can provide, the above major advantages make interactive

procedures increasingly more important in multiple criteria decision making (MCDM). In the last two decades there has been an extensive interest in developing interactive approaches for MOLP (Benayoun, deMontgolfier, Tergny and Laritchev, 1971; Benson and Aksoy, 1991; Fichefet, 1976; Grauer, Lewandowski and Wierzbicki, 1984; Masud and Hwang, 1981; Reeves and Franz, 1985, Spronk and Telgen, 1981; Steuer, 1977; Steuer and Choo, 1983; Zionts and Wallenius, 1983). Recently, Aksoy (1990) surveyed the literature dealing with interactive multiple objective decision making from 1965 to 1988.

1.2 An Overview of The Problem of Optimizing a Linear Function over the Efficient Set

The problem (P) of optimizing a linear function over the efficient set can arise in a variety of situations where a linear function is available which acts as a criterion for measuring the importance of the efficient alternatives that are available. The problem (P) of optimizing a linear function over the efficient set for a MOLP may be written

$$\text{Max} \quad \langle d, x \rangle \qquad \text{(P)}$$

$$\text{subject to} \quad x \in X_E,$$

where X_E is the set of efficient solutions for problem (MOLP), and $d \in R^n$.

Notice with $d = -c$, where c is one of the elements of $\{c_1, c_2, \ldots, c_p\}$, that an important special case of problem (P) involves finding a minimum criterion value over the efficient set X_E of problem (MOLP). This problem may be written

$$\begin{array}{ll} \text{Min} & \langle c, x \rangle \\ \text{subject to} & x \in X_E, \end{array}$$

where X_E is the set of efficient solutions for problem (MOLP) and c is any element of $\{c_1, c_2, \ldots, c_p\}$.

In general, when X is nonempty, X_E is a nonconvex set (Benson, 1984). Therefore problem (P) is generally a nonconvex programming problem. In nonconvex programming problems, standard convex optimization techniques generally fail. This is due to the existence of local optima that are not global. Because of this difficulty, the methods devised for analyzing nonconvex programming problems are quite diverse and significantly different from standard methods. A rapidly-growing number of methods have been developed for solving specific classes of nonconvex programming problems (see e.g. Al-Khayyal and Falk, 1983; Benson, 1985; Falk and Soland, 1969; Horst, 1976, 1988; Horst and Tuy, 1990 and references therein; Pardalos and Rosen, 1987).

There are important reasons why the problem (P) needs to be considered. Benson (1984, 1986, 1990) pointed out that by solving problem (P), the computational burden of generating the entire efficient set is avoided. This is potentially quite beneficial, since the computational burden of generating this set grows rapidly with problem size. Furthermore, the DM is not required to choose a preferred solution from a potentially overwhelmly large set of efficient solutions.

Some applicable situations for problem (P) have been discussed by Benson (1984, 1986). In these articles, Benson points out that one important practical situation in which a linear function is available for discriminating among efficient points is when one seeks to find the range of values that a criterion function $<c_i, x>$ of problem MOLP takes over the efficient set.

1.3 An Overview of Citrus Rootstock Use in the Citrus Industry

Citrus trees generally begin bearing fruit within 3 to 4 years of planting, depending upon the growing region. They may remain productive for more than 30 years under commercial conditions. Commercial citrus species, which include sweet oranges, tangerines, grapefruit, lemons and limes, are produced in over 50 countries. The progenitor of citrus probably originated somewhere in West Gondwanaland nearly 140

million years ago (Davies, 1987-1988). Up to that time the major continents as we know them today were divided into two supercontinents, Gondwanaland (South America, Africa and Australia) and Laurasia (North America and Eurasia). The time and distance involved in creating today's continents created an interesting distribution of citrus variety and species. For example, citrus species found in Australia (Eremocitrus and Microcitrus) are quite different from those originating in India (sweet orange), but similar to those which originated in the Malay Archipelago (limes, lemons).

The origin of citrus and its relatives is of particular interest to citrus breeders. Walter T. Swingle, one of the pioneers in citrus breeding, was the first to breed cold hardiness into citrus rootstocks. Following the devastating freeze of 1894-95, he decided to cross Poncirus trifoliata, a cold tolerant species which originated in North China, with Citrus sinensis, the sweet orange. The resulting hybrid produced Carrizo and Troyer citranges, the former being one of the most widely used rootstock in Florida and the latter the primary rootstock for California.

Seedlings were common in much of the world until Phytophthora disease appeared in the Azores, located in the Mediterranean, in 1842. As a result, the transition of citriculture from seedling to budded trees began (Chapot, 1975). Phytophthora is the fungus which causes foot rot, which is a disease of the bark on the lower trunk or crown

roots of citrus. As Phytophthora spread and became recognized, interest in rootstocks greatly increased because of the experienced tree loss among the seedling trees. Phytophthora was later noted in all of the Mediterranean countries, and by about 1935, it had been observed nearly everywhere (Chapot, 1975). By recognizing that rootstocks provide certain advantages that are beneficial to a citrus tree (Castle et al., 1989), seedlings were gradually replaced, so that today virtually all trees are propagated by budding onto rootstocks.

During propagation, usually two genetically different plant materials are combined to form the citrus tree. The relationship between scion and rootstock is of fundamental importance to successful long-term commercial performance. Scion is the portion of the citrus tree which produces the desired fruit and arises from the bud inserted in the rootstock seedling. Within each citrus species there are a number of scions that are further separated based on fruit, tree characteristics, and harvest season. For example, within sweet orange, there are a number of scions, which are Valencia, Hamlin, Parson Brown, Navel, Pineapple and Temple. Red Blush and Marsh are two commonly used scions within grapefruit.

When the union between scion and rootstock takes place readily and the tree continues to grow and develop without difficulty, it is said to be a compatible union. Experience

in Florida has demonstrated that the commonly used scions in all citrus species are compatible with most citrus rootstocks.

More than 80% of the citrus trees in Florida are sweet orange. The two most widely used scions in sweet orange are Valencia and Hamlin as shown in the Table 1. The data in Table 1 are taken from the Bureau of Citrus Budwood Registration.

Table 1. Registered citrus tree propagations in Florida by variety as a percentage of each group of total sweet oranges from 1981 to 1987

Scion Variety	Year 81/82	82/83	83/84	84/85	85/86	86/87
Valencia	38.3	30.6	37.2	42.0	35.1	43.1
Hamlin	45.3	53.8	46.8	42.5	48.5	32.7
Pineapple	5.8	4.5	5.5	3.2	2.9	7.0
Navel	8.4	8.2	6.9	4.9	6.5	9.3
Parson Brown	0.6	2.5	2.8	6.5	6.6	6.4
Temple	1.6	1.1	0.8	0.9	0.4	1.5

Valencia sweet orange is in high demand by processors because it can be blended with lower quality juices to insure uniform quality. Moreover, pounds-solids/box (defined in Section 4.1-2) for Valencia averages 6 to 7 versus 4 to 5 for Hamline. Consequently, dollar returns have been very high

for Valencia over the past few years. Hamlin sweet orange is a major sweet orange scion grown in Florida mainly because of its high productivity and regularity of fruiting.

There are many rootstocks, and each has a wide range of performance characteristics that affect the scion budded on it. There are also scion effects on various rootstock characteristics, but these effects are not as well-known or understood as those of the rootstock.

Citrus rootstocks affect more than 20 horticultural and pathological characteristics of the tree and fruit. Each rootstock has its own horticultural characteristics which have pronounced effects on tree vigor and size, fruit yield and size, juice quality, and tolerance to cold, drought, flooding, and salt. Also, each citrus rootstock has its own pathological characteristics which have significant affects in tree tolerance to diseases such as blight, tristeza and Phytophthora.

Phytophthora spawned a search for resistant rootstocks, and Sour orange became dominant. However, difficulty was encountered with this rootstock in South Africa and Australia. Trees declined within several years after planting. As a result, Rough lemon became popular in both countries. In 1946, it was reported that the decline of trees on Sour orange was presumably caused by a tristeza disease. Tristeza is a citrus virus disease which is transmitted by an insect. It was first recognized in Florida in about 1950. In the

years since, it has been spread by aphids and has infected nursery stock in every citrus growing area of the state. In fact, 95.5% of all citrus in Florida has some form of tristeza. However, Sour orange rootstock is the only rootstock causing tree decline in Florida citrus from tristeza. Thus the two diseases, Phythophthora and tristeza, considerably accelerated rootstock use and development. Recently, a newly found disease, citrus blight, has rendered millions of trees unproductive in Florida and elsewhere in the world. Citrus blight, the cause of which is yet unknown, is the most serious of a number of problems causing tree decline in Florida citrus. Satisfactory control measures for blight have yet to be discovered.

Especially because of tristeza and blight, efforts have intensified to develop new rootstocks which are resistant to these diseases. New rootstocks are the result of selection and breeding procedures. Classical breeding involves pollination and hybridization. Recently, new techniques such as fusing cells and tissue culture to regenerate whole plants have been used. These new techniques are faster and may circumvent some of the obstacles encountered in the classical approach. The development of a new rootstock, however, remains an inherently long process, since horticultural field evaluations are required. The entire process can easily take 15 years or longer (Castle et al., 1989). For example, Swingle citrumelo, which is a relatively new rootstock in

Florida, is currently finally gaining widespread popularity in Florida and Texas after over 80 years of development and evaluation.

Since no single rootstock is perfect for all situations, the selection of rootstocks is a major consideration in every citrus operation. It is fundamental to the success of the mature grove, since the rootstocks chosen will become the root system of the budded tree.

1.4 Organization of the Dissertation

Literature surveys of the problem of optimizing a linear function over the efficient set, of interactive algorithms for MCDM, and of the use of citrus rootstock in Florida are given in Chapter 2.

In Chapter 3, computational experience with the Benson-Sayin heuristic algorithm for problem (P) is reported. Also included in Chapter 3 are discussions concerning the range of compromise and the use of payoff table solutions in MOLP procedures.

In Chapter 4, the citrus rootstock selection problem in Florida is stated and a MOLP model is developed for this problem. This model is formulated in a general form, so that it can be used at any citrus enterprise in Florida if the necessary data are available. The model is applied to a

certain real-world citrus rootstock selection problem in the Fort Pierce area, which is located in St. Lucie county in southeast Florida. From the obtained data, the coefficients of the objective functions and the constraints are estimated. To solve this specific citrus rootstock selection problem at Fort Pierce area, the interactive STEM-method (Benayoun et al., 1971) is chosen from among the various procedures suited for this problem (see Section 4.2.2).

The citrus rootstock selection problem is solved in two different ways, each using the STEM-method. One way uses payoff table solutions and the other uses Benson-Sayin heuristic solutions. The results from the two approaches are compared and analyzed in order to see possible differences in the efficiency of the STEM method when implemented in these two different ways. To assess these possible differences, the quality of the final solution, the total number of iterations required to find this solution, how well the DM responds in choosing his aspiration levels for the objective function values at each iteration, and how well the required minimax weights (see Section 2.2) are calculated are all examined. These comparisons between the two approaches showed that the efficiency of the STEM method improves, at least when solving the citrus rootstock selection problem, from the use of the Benson-Sayin heuristic method.

Finally, Chapter 5 gives the conclusions and a summary of the dissertation

CHAPTER 2

LITERATURE REVIEW

Research on the problem (P) of optimizing a linear function over the efficient set, which contains as a special case the problem of finding a minimum criterion value over the efficient set, is reviewed in Section 2.1. In Section 2.2, the literature on interactive algorithms for the problem (MCDM) is reviewed and some of the popular interactive algorithms for problem (MOLP) are presented. This will be followed in Section 2.3 by a survey of the literature on citrus rootstocks in Florida.

2.1 Literature Survey of the Problem of Optimizing a Linear Function over the Efficient Set

In spite of the potential benefits (see Section 1.2) which can be obtained by optimizing a linear function over the efficient set, relatively few attempts have been made to solve problem (P). This is probably at least partially due to the inherent difficulties involved in solving this global optimization problem.

Problem (P), which is the problem of optimizing a linear function over the efficient solutions of multiple objective linear program (MOLP), was first proposed by Philip (1972). He presented an outline of a procedure which uses cutting planes to solve the problem (P). This procedure attempts to find a globally optimal solution for problem (P). It is based on the fact that the set of efficient points of a polyhedron is connected (see Steuer, 1986), and on the fact that at least one optimal solution for problem (P) is an extreme point of X (see Benson (1984)). However, there are considerable difficulties in implementing this procedure. These difficulties are mainly due to the following two weaknesses in the algorithm.

First, whenever a cutting plane restriction is added, this algorithm requires searching for all new extreme points created by the original feasible set X and the added hyperplane. In the algorithm, it is not clearly stated how to implement this search. Second, even for a small problem, we may need to add many cutting plane restrictions.

More recently, Benson (1990) presented the first readily implementable algorithm for problem (P). His algorithm is a relaxation algorithm for finding a globally optimal solution for problem (P). This algorithm can be implemented using only linear programming methods. However, it can be computationally burdensome to find an optimal solution using this algorithm because of the large number of branchings that

may be needed to execute the branch and bound procedure called for in the algorithm. In his article (1990), Benson proved that his algorithm always terminates with an exact optimal solution to the problem after a finite number of iterations. He also presented the possible uses of the knowledge of the range of values which can be obtained by finding the minimum criterion values over the efficient set.

Dessouky, Ghiassi and Davis (1986) developed three heuristic procedures for minimizing any criterion of a multiple objective linear program over its set of efficient solutions. These three heuristic procedures are a simple pivoting procedure, a constrained pivoting procedure, and a bilinear search procedure.

In the beginning of each of these three procedures, the following two steps need to be performed. First, the existence of a special case, in which the minimum criterion value over the efficient set is the same as the minimum feasible value, is explored. If it is detected that the minimum criterion value over the efficient set is the same as the minimum feasible value, then the procedures stop. Second, an initial efficient extreme point is selected.

For discussion purposes, let x^o denote the initial efficient point found by each algorithm, and assume that the kth minimum criterion value over the efficient set needs to be estimated. The simple pivoting procedure, starting with x^o and using multicriteria simplex tableaus, pivots from one

efficient extreme point to a neighboring one with either a smaller or equal value in the κth criterion until no more such points can be found.

The constrained pivoting procedure, which also has a simplex pivoting procedure imbedded within it, occasionally also introduces an additional constraint of the form $<c_k,x> \leq q_k$, where $q_k = <c_k,x^*>$ and x^* is the current efficient extreme point. Moving from an efficient point along one of the edges of the hyperplane $<c_k,x>=q_k$ may yield another efficient point of X which is not adjacent to x^*. Whenever an efficient point with a smaller value of the κth criterion is found, the value of $<c_k,x>$ is updated. The additional constraint may be left unchanged as long as new, adjacent efficient points with either smaller or equal values of the κth criterion are found. If such efficient points can not be found and the slack of the additional constraint is not equal to zero, then the value of q_k is set equal to q_k , where q_k is the last value of $<c_k,x>$ achieved. This permits pivoting to a new basis and moving along an edge of the hyperplane $<c_k,x>=q_k$ to another efficient extreme point of X. However, if such efficient points can not be found and the slack of the additional constraint is equal to zero, the procedure stops.

The bilinear search procedure involves solving a certain quadratic programming problem, which is not convex, at each iteration.

In their article (1986), Dessouky et al. tested the performance of these three procedures on three small examples (three objective functions and three variables). These tests showed that both constrained pivoting and bilinear search offer better solutions than simple pivoting. However, it is unknown how effectively these procedures perform, in general, in estimating the minimum criterion value over the efficient set, since no additional computational results are available.

The problem addressed by Dessouky et al., which is a special case of problem (P), is motivated by the following two facts. First, decision makers often desire to determine the ranges of values the criteria take over the efficient set, and knowledge of the minimum values of the criteria over the efficient set are essential to determine these ranges. Second, decision makers realize that the wide use of estimates in lieu of the minimum values over the efficient set are misleading.

Benson (1986) developed an algorithm for solving the problem of optimizing a linear function over the set of weakly efficient solutions of multiple objective linear program. In this algorithm, an optimal or approximately optimal solution for the problem is found in a finite number of steps. He also showed some computational experience which indicates that the algorithm is quite practical for relatively small problems.

Recently, Isermann and Steuer (1987) outlined three conceptual approaches for computing the minimum criterion values over the efficient set.

The first is to use a vector-maximum code such as ADBASE (Steuer, 1983) or EFFACET (Isermann, 1984) to compute all efficient extreme points. The minimum criterion values over the efficient set are determined by examining the components of the criterion vectors of each of the efficient extreme points. However, the amount of computer time required for the vector-maximum generation of all efficient extreme points can be too large for this approach to be a serious candidate for solving problem (P).

Another approach suggested by Isermann and Steuer involves solving a certain primal-dual feasible program. The difficulties with this approach are the relatively large size of the primal-dual feasible program and the involvement of a nonconvex quadratic constraint.

The third approach outlined by Isermann and Steuer is a simplex-based procedure, which is essentially Philip's cutting plane algorithm, but applied to the minimum criterion case.

In a more theoretical vein, Benson (1984) has studied properties of the problem of optimizing a linear function over the efficient set of a multiple objective linear or nonlinear program. In this article, for both the case when the feasible set X is a closed convex set and when the feasible set X is a polyhedral convex set, Benson examined the nature of an

optimal solution and of the optimal solution set. He also developed necessary and sufficient conditions for the problem to be unbounded. From Corollary 4.2 in his article (1984), Benson showed that whenever X is a polyhedral set in $\mathbb{R}^2$ and a certain nondegeneracy assumption holds, an optimal solution can be found by simply comparing all optimal extreme point efficient solutions for the linear programming problems (R) and (P_i), $i = 1,2,...,p$, where the problem (R) is given by $\max_{x \in X} \langle d,x \rangle$ and, for each $i = 1,2,..,p$, the problem (P_i) is given by $\max_{x \in X} \langle c_i,x \rangle$.

2.2 Literature Survey of Interactive Algorithms for Problem (MCDM)

Because of the advantages of the interactive methods (see Section 1.1.2), many knowledgeable individuals in the field of MCDM would agree with Steuer's (1986, p. 361) statement that "the future of multiple objective programming is in its interactive application."

Many of the interactive algorithms have been developed for the problem (MOLP) (Benayoun, deMontgolfier, Tergny and Laritchev, 1971; Fichefet, 1976; Grauer, Lewandowski and Wierzbicki, 1984; Masud and Hwang, 1981; Reeves and Franz, 1985; Spronk and Telgen, 1981. Steuer, 1977; Steuer and Choo, 1983; Zionts and Wallenius, 1983). Several algorithms have also been developed for solving multiple-objective nonlinear and integer programming problems (Geoffrion, Dyer and

Feinberg, 1972, Montgomery and Bettencourt, 1976; Zionts and Wallenius, 1976; Musselman and Talavage, 1980; Ross and Soland, 1980; Friesz, 1981; Marcotte and Soland, 1981). More recently, Aksoy (1990) surveyed the literature dealing with interactive multiple objective decision making from 1965 to 1988.

It is unlikely that any single procedure will emerge as universally preferred because different procedures may be better suited for different types of DMs and decision making situations. Among these solution procedures, different magnitudes of requirements are placed on the DM in terms of both the quality and quantity of information required of him. Because of these different capabilities and requirements, experiments designed to compare existing interactive procedures are often performed.

For instance, Buchanan and Daellenbach (1987) have investigated the relative performance of four different solution methods from the point of view of the user of the method. These methods are the method of Zionts and Wallenius (1983), the surrogate worth tradeoff method of Haimes and Hall (1974), the Tchebycheff procedure of Steuer and Choo (1983), and a "naive" method of Buchanan(1985). Their experiment showed that the method of Steuer and Choo (1983) was clearly preferred over all methods. However, they concluded that the ultimate solution method will be a hybrid approach which can adequately accommodate the different decision making

strategies and behavior of different decision makers. They also stressed that further experiments are necessary to examine the validity of their results with respect to other types of solution methods and other types of decision problems.

In the remainder of this section, some of the more popular interactive algorithms for solving problem (MOLP) are briefly reviewed. These algorithms could be used to solve the citrus rootstock selection problem developed in this dissertation. For more complete surveys, see Evans (1984) and Steuer (1986)

The Step Method (STEM) (Benayoun et al., 1971) was one of the first techniques developed to address multiple-objective linear programming problems via an interactive approach. STEM employs a single objective model which minimizes the maximum weighted distance of the problem's objective function values from the ideal criterion vector (see Section 3.1) values by using minimax (Tchebycheff) weights. The set of constraints for this single objective problem is identical to that of the original multi-objective problem in the first iteration. In STEM, at first, a payoff table is computed and is presented to the DM in order to demonstrate to him the ranges between the best and the worst objective function values. However, due to certain payoff table difficulties, these ranges can be incorrect (see Section 3.1). From these ranges weights are computed in order to construct an auxiliary

objective function by adding the weighted original functions. This auxiliary objective function is then used to generate an initial solution. If the DM is not content with the value of one or more of the objective functions, he must be willing to relax the value of at least one of them. He also must specify which objective functions he is willing to relax and by how much he is willing to relax each one. In each subsequent iteration, the DM is also asked to adjust the feasible region by adjusting his aspiration levels for the objective functions in a similar way. The solutions generated by STEM are not restricted to the extreme points of X. These solutions are weakly efficient points (see Sawaragi et al., 1985). This allows the DM to explore points in the efficient set, since the weakly efficient set includes the efficient set. Although STEM is an ad hoc procedure, it is easy to understand and is flexible. These characteristics make it a good candidate for use as a part of a large decision support system.

The weighted Tchebycheff procedure (Steuer and Choo, 1983) is a weighting vector space reduction method. This procedure first computes an ideal criterion vector. It then uses this ideal vector throughout the procedure to generate efficient solutions which minimize a weighted distance measure from that ideal. The first iteration begins by forming a dispersed group of weighting vectors. The efficient solutions are computed by solving the augmented (or

lexicographic) weighted Tchebycheff program for each of the weight vectors. Then, the DM is asked to choose his most preferred criterion vector from the generated efficient solutions. At the second iteration, another set of weighting vectors is formed, but this time, more concentrated than the first set, and centered around the weight vector corresponding to the DM's most preferred criterion vector at the first iteration. Using the new set of weighting vectors, the efficient solutions are computed by solving the augmented (or lexicographic) weighted Tchebycheff program for each of the weight vectors. The DM is asked to choose his most preferred criterion vector from the newly computed efficient solutions. In this way, at each iteration, a certain fixed number of efficient solutions in the objective function space are offered to the DM. These solutions are calculated in such a way that they are maximally dispersed. The DM has to choose which solution is preferred. Then, using this solution, a new weighting vector is calculated. This new weighting vector is then used to compute a new set of weighting vectors, which is smaller than the set of weighting vectors used in the previous iteration. As the iterations proceed, the weighted Tchebycheff procedure provides the DM with an increasingly concentrated set of solutions. Although it is especially suitable for problem (MOLP), the procedure is also applicable to nonlinear multiple-objective programs with the use of suitable software.

Zionts and Wallenius (1983) have developed a reduced weighting vector space method for problem (MOLP) assuming an unknown pseudoconcave value function. The method is an extension of an earlier work (Zionts and Wallenius, 1976). The method operates by iteratively asking the DM questions about adjacent extreme points or tradeoff vectors. From the responses, portions of the initial weighting vector space are eliminated The process continues until the weighting space has been reduced to a small enough region for a final solution to be identified.

In the pairwise-composition method (Kok and Lootsma, 1985), payoff matrix information in each iteration is used. By showing the ideal and nadir vector, the DM is requested to make pairwise comparisions in order to estimate the ratios which are acceptable for deviations from the ideal vector at each iteration. Then, using the obtained ratio, the nearest feasible solution using the Tchebycheff norm is found. The algorithm iteratively presents the DM with this feasible solution and asks for the specification of relaxation quantities for the objective function values.

With the recognition that not enough attention has been paid to the design of user interfaces, Korhonen and Wallenius (1988) developed a dynamic, visual, interactive procedure for multiple-objective linear programming (MOLP), which is called a Pareto race. The foundation of Pareto race originates in the visual reference direction approach to the MOLP problem

developed by Korhonen and Laakso (1986a, 1986b). In the original procedure, using a reference direction, a subset of efficient solutions is generated and presented for the DM's evaluation. The interface is based on a graphic representation, but it is static by nature. One picture is produced at each iteration. However, Pareto race improves upon this procedure by making it dynamic. In Pareto race, a constraint is regarded as a subset of goals, which is referred as an inflexible goal in the MOLP problem. In each iteration, the DM is asked to evaluate the values of the flexible and inflexible goals and of the decision varibles. If he is satisfied, the algorithm stops. Otherwise, the DM can adjust the aspiration level for each goal. Depending upon his aspiration level for each goal, the method updates the reference direction. The process continues until the DM is satisfied with the solution found. During the process, a decision maker can freely search the efficient frontier of the MOLP problem by controlling the speed and direction of motion. On a display, the DM sees the objective function values in numerical form and as bar graphs whose lengths dynamically change as he moves about on the efficient frontier. This allows the DM to feel that the system is entirely under his control.

2.3 Literature Survey of Citrus Rootstocks in Florida

Citrus is one of Florida's most important agricultural crops. The on-tree value (the value of the fruit after subtracting harvesting costs) of all citrus for the 1987-88 season was an estimated $1.3 billion, about 27% above the previous record of $1.03 billion in 1979-80 (Jackson et al., 1989). Florida has produced more than 70% of the total U.S. citrus production during each of the past 20 years, averaging over 9 million tons of fruit per year. The single most significant discovery in the citrus industry during the last 90 years was the development of frozen concentrate in the late 30s and early 40s. Currently 85% of Florida citrus is processed.

Historically, the major rootstocks of the citrus industry in Florida have been Rough lemon and Sour orange. However, the prevalence of blight in recent years has severely limited the use of Rough lemon, and the increasing incidence of citrus tristeza virus is significantly affecting the use of Sour orange, as shown in Table 2 in next page. The source of the data in Table 2 is the Citrus Budwood Registration Program for Florida. As the data in Table 2 indicate, Rough lemon is no longer used because of its susceptibility to blight.

A rootstock such as Rough lemon can decrease in commercial use within a short-time period, while the

development of a new rootstock is inherently a much longer process (see Section 1.3). Furthermore, it is unlikely that a new rootstock will have only desirable attributes. The Florida citrus industry is constantly changing, and rootstock research is frequently directed to improving those rootstock traits of the highest priority. A new rootstock that is intolerant of tristeza virus or blight but is otherwise above average would probably be of limited commercial value.

Table 2. Rootstocks used for registered and validated nursery trees 1965-89 (%)

Year	Sour orange	Rough lemon	Cleopatra mandarin	Carrizo citrange	Swingle citrumelo	Miscellaneous
65-66	49.7	29.8	14.3	2.3	-	3.9
66-67	44.2	32.6	11.8	0.7	-	10.7
67-68	34.7	49.2	4.1	6.3	-	5.7
68-69	32.7	39.4	8.3	13.8	-	5.8
69-70	26.9	38.7	8.3	23.1	-	3.0
70-71	29.4	35.9	7.1	23.0	-	4.6
71-72	41.9	12.9	19.4	16.6	-	9.2
72-73	41.9	1.5	11.9	29.1	-	15.6
73-74	39.0	0.4	11.8	34.6	-	14.2
74-75	32.1	0.6	9.5	46.2	1.6	10.0
75-76	27.0	2.1	7.9	38.0	6.9	18.1
76-77	18.9	1.9	8.0	51.5	3.5	16.2
77-78	21.4	1.0	13.1	40.2	6.9	17.4
78-79	25.2	1.7	10.0	40.1	5.8	17.2
79-80	20.1	-	13.1	41.1	12.4	13.3
80-81	28.2	0.3	6.2	45.4	11.4	8.5
81-82	31.4	-	6.9	40.6	10.2	10.9
82-83	38.1	-	10.0	33.8	11.8	6.3
83-84	32.6	-	8.3	38.0	15.2	5.9
84-85	28.5	-	8.3	28.6	30.1	4.5
85-86	33.0	-	18.2	25.5	18.6	4.7
86-87	31.0	-	19.9	35.2	6.7	7.2
87-88	6.0	-	22.9	34.4	31.0	5.7
88-89	4.6	0.1	18.8	19.4	50.2	6.9

During the 1970s and 1980s, rootstocks became a more critical issue than in previous years in Florida, largely because of blight, the increased incidence of tristeza, and the increased frequency of freezes (Castle et al., 1989). These three factors have contributed greatly to reducing the expected life of a grove. Besides these major limiting factors, the widespread occurrence of Phytophthora fungi pathogenic to citrus continues to have a significant influence on the rootstock situation in Florida. Even though much can be done to prevent serious infection by the adoption of improved cultural practices and the use of suitable preventive measures such as fungicides, it costs a great deal to perform these preventive measures on a regular basis. Moreover, using fungicides may raise the questions concerning the environmental safety.

One of the most important factors to be considered in selecting rootstocks is drought tolerance. Drought tolerance has decreased in importance in rootstock selection, particularly because of the widespread use of irrigation which can mask rootstock differences. However, increasing regulation may affect the future supply and use of water for agriculture in Florida. This and other factors may thus cause rootstock drought tolerance to be reexamined.

The first review of citrus rootstocks was published in 1948 (Webber, 1948) and a second review followed in 1979

(Wutscher, 1979). Recently, Castle (1987) gave a comprehensive review of citrus rootstocks.

One of the earliest Florida publications about citrus rootstocks was published in 1955. Jasper Joiner (1955) stated that rootstocks "can mean the difference between success and failure in a grove operation." Sour orange, Rough lemon, Sweet orange, Cleopatra mandarin, Trifoliate orange, Rusk citrange and Rangpur were described in four pages. Fred Lawrence and Don Bridges (1974) expanded the discussion to include Carrizo citrange plus several rootstocks which had been recently released. Interest in rootstocks has broadened and intensified greatly since the 1974 circular was published. Research on citrus rootstocks has been accelerated by the frequent freezes and the increased incidence of tristeza and blight in Florida. Many attempts have been made to study the tolerance of each rootstock to the following important factors: (1) Freeze (Yelenosky, 1976; Yalenosky and Young, 1977; Yelanosky et al., 1981; Yalenosky et al., 1984; Rouse et al , 1990); (2) Blight (Young et al., 1980, 1982; Youtsey, 1986), (3) Tristeza (Garnsey et al., 1980; Yokomi and Garnsey, 1986; Brlansky et al, 1986); (4) Phytophthora (Hutchison and Grimm, 1972; Smith et al., 1987).

In 1988, D. Hutchison summarized in a general way, across all scions, the relative ranking of various rootstocks for some important characteristics. This summary is presented in Table 3 in next page.

In Table 3, Fruit yield and fruit solids represent quantity and quality, respectively (see Section 4.1.2).

The high susceptibility of Rough lemon to blight has virtually eliminated its use in new plantings in Florida (see Table 1 and Table 2). However, if grove removal for replanting is anticipated within about ten years from the initial planting, Rough lemon is ideal for resetting because of its strong performance as a young tree.

Table 3. Summary of important rootstock characteristics
1 = best 5 = poorest.

Rootstock	Blight Tolerance	Tristeza Tolerance	Cold Hardiness	Fruit Yield	Fruit Solids
Cleopatra mandarin	2	1	2	3	2
Rough lemon	5	1	5	1	4
Swingle citrumelo	2	1	2	3	3
Carrizo citrange	3	1	3	2	3
Sour orange	2	5	2	2	2

Carrizo citrange has attained widespread use because of high yields and fruit quality as well as tolerance to tristeza virus and Phythophthora. Even though it is blight

susceptible, the attributes of Carrizo citrange appear to compensate for its disadvantages. This clearly illustrates the justifiable value that growers place on yield and juice quality. No rootstock is immune to tree loss from blight, but trees on Sour orange, Cleopatra mandarin and Swingle citrumelo are the least affected. However, there are different compromises and risks associated with these rootstocks.

Sour orange, which produces relatively moderate yields of excellent quality fruit, has one glaring weakness, high sensitivity to tristeza virus. Increasing problems with tristeza have seriously diminished industry interest in Sour orange. Sour orange use might be justified for replanting in mature groves on Sour orange where little or no loss has occurred from tristeza virus. In the northern areas of the citrus industry and in any relatively cold sites, cold tolerance becomes a principle limiting factor. Sour orange might be an appropriate choice after careful evaluation of local tristeza conditions. However, Cleopatra mandarin and Swingle citrumelo are other suitable choices that make Sour orange a questionable risk.

Cleopatra mandarin has been used in Florida for many years where it has been and remains an excellent rootstock. Cleopatra mandarin has blight, cold and tristeza tolerance. Because of relatively low yields, Cleopatra mandarin is an excellent example of the value placed on yield. It has many

fine traits, but they apparently do not surpass the importance of productivity when growers put high priority on maximizing profits within a short-term period and less priority on minimizing tree damage. However, there is an increased interest in Cleopatra mandarin because in those groves where blight loss is heavy, Cleopatra mandarin may be the best choice for replants. Trees on Cleopatra mandarin may be slow to reach their full bearing potential, but cumulative yield in conjunction with tree survival must be balanced against young tree behavior.

The current success of Swingle citrumelo in Florida (see Table 2) can be attributed to its many desirable characteristics, particularly tristeza tolerance and its apparent blight tolerance to date. However, Swingle citrumelo is relatively slow growing.

In 1989, a considerably expanded and more comprehensive publication (Castle et al., 1989) for the rootstocks of Florida citrus was published. This publication included detailed discussions of rootstock characteristics, strengths, weaknesses and suggested uses as well as selection strategies

CHAPTER 3

MINIMUM CRITERION VALUES OVER THE EFFICIENT SET

Minimum criterion values over the efficient set are of interest in multiple objective mathematical programming in order to characterize the ranges of the criterion values over the efficient set. Knowledge of the range of values of a criterion function over the efficient set has various potential uses in MOLP applications (see Benson, 1991). This range of values is referred to as the range of compromise. In fact, the range of compromise has special significance which stems from the following considerations:

1. The range of compromise provides the DM with insight into selecting parameter values such as goals or aspiration levels.

2. Usually not all of the objective functions in a multiple objective problem are of equal importance. The range of compromise can be utilized for ranking the objective functions. For example, if the range of compromise is relatively small for a given objective function, achievement of an acceptable level for that objective will likely not require a corresponding high ranking.

3. The number of objective functions in a multiple objective problem is an influential factor in the computational effort needed by techniques used to solve the problem. One way to reduce the computational burden is to transfer an objective function from the objective function set to the constraint set. The range of compromise can be used as an indicator for deciding whether or not a given objective function can be transferred from the set of objective functions to the set of constraints. If the range of compromise is relatively small, the objective function can be transferred to the constraint set and bounded by the endpoint values for its compromise range.

The range of compromise may also be needed in practical problems. For instance, Ghiassi and Dessouky (1984) applied the concept of the range of compromise in multiple criteria decision making to certain metal cutting problems.

However, in general, it is not easy to find the range of compromise for a given objective function $\langle c_i, x \rangle$ of problem (MOLP), since determining the minimum value of i-th objective over the efficient set is a highly-complex problem. The difficulties are that the efficient set X_E is not known explicitly and is, in general, a nonconvex set. Because of these difficulties, minimum criterion values from payoff tables (defined in Section 3.2) have often been used in multiple objective linear programming.

In this chapter, the range of compromise and the use of payoff table solutions in MOLP procedures are discussed. Then our computational experience with the Benson-Sayin heuristic procedure for problem (P) is reported.

3.1 The Use of Payoff Table Solutions in MOLP solution Procedures

Because of the lack of an efficient procedure for determining minimum criterion values over the efficient set, estimates of minimum criterion values using payoff tables have often been used in the MOLP problem to attempt to provide bounds on the values of each of the objective functions over X_E. To define the minimum criterion value estimates found by using a payoff table, let x^i denote the solution resulting from the i-th individual objective function maximization over X. Then, a payoff table is constructed as follows:

Payoff table

c_1x^1	c_2x^1	. . .	c_px^1
c_1x^2	c_2x^2		c_px^2
.			
.			
.			
c_1x^p	c_2x^p		c_px^p

The estimates of the minimum criterion values using the payoff table are defined as the values of $\min_{1 \le j \le p} (c_i x^j)$, i =

$i=2,\ldots,p$. The $c_i x^i$ entries along the main diagonal form the ideal criterion vector.

Among the interactive solution procedures that utilize payoff table information are the STEM method of Benayoun, deMontgolfier, Tergny and Laritchev (1971), the Interactive Sequential Goal Programming method (ISGP) of Masud and Hwang (1981), the Ellipsoidal Interactive Multiple Goal Programming method of Spronk and Telgen (1981), the Pairwise-composition method of Kok and Lootsma (1985), and the Simplified Interactive Multiple Objective Linear Programming (SIMOLP) method of Reeves and Franz (1985).

Weistroffer (1985) pointed out that the potential users of these methods should be warned that such payoff table minimum values are not necessarily equal to lower bounds for the objective function values over the efficient solution set. In addition, he indicated that any solution technique should allow the user to also investigate solutions with objective functions values lower than the payoff table minimum values.

MOLP procedures which utilize the payoff table solutions may overlook elements of the set of efficient solutions. For example, in the STEM method (Benayoun et al., 1971), the ranges of the criterion values over the efficient set may be either understated or overstated because of the use of payoff table solutions. If these ranges are incorrect, the minimax weights which are required to be calculated in the STEM method would be incorrectly specified. This could result in

reducing the efficiency of the STEM method from various points of view. For instance, this could reduce the ability of the DM to explore the weakly-efficient set, reduce the quality of the final solution, increase the total number of iterations required to find this solution, and decrease the ability of the DM to respond to tradeoff questions.

Isermann and Steuer (1987) report computational experience concerning the degree to which the payoff table minimum values might furnish good or bad estimates of the lower bounds for the objective function values over the efficient set. In their article (1987), they showed that the field of multiple objective programming needs a better method than the payoff table minimum values for estimating the minimum criterion values over the efficient set. They reported computational experience that demonstrates that the discrepancies between the payoff table minimum values and the minimum criterion values over the efficient set can often be large.

Recently, Reeves and Reid (1988) also examined the difference between the use of payoff table estimates of and actual values for the minima of the criteria over the efficient set by performing some computational experiments. In their experiments, payoff table solutions from lexicographic maximization are used in order to ensure that these solutions are efficient. Their experiments are similar to those reported in Isermann and Steuer (1987), but

lead to some differences in the interpretation of results. They conclude that there usually exists a sufficient number of efficient solutions within the range of criterion values obtained from the payoff table solutions to afford decision makers a reasonable and representative set of choice in most decision making situations without additional computational effort.

It is noteworthy that there is a tradeoff between the relative computational ease of the solution procedures utilizing payoff table solutions and the possibility of overlooking some members of the set of efficient solutions.

3.2 Computational Experience with the Benson-Sayin Heuristic Algorithm

More recently, Benson and Sayin (1991) developed a heuristic algorithm for solving problem (P). Their study is motivated mainly by the following three facts. First, to get an exact optimal solution for problem (P) is computationally burdensome (see Section 2.1). Second, currently available heuristic algorithms proposed by Dessouky et al. (1986) are only applicable to the special case of problem (P) and the practical use of these algorithms is unknown (see Section 2.1). Finally, payoff table methods are not reliable methods in estimating minimum criterion values over the efficient set (see Section 3.1).

The Benson-Sayin heuristic algorithm searches some of the efficient extreme points of X by generating and searching a sample of efficient faces in criterion space. It can be implemented using only linear optimization methods. Since finding the minimum criterion value over the efficient set is a special case of problem (P), we can use the Benson-Sayin heuristic method to estimate minimum criterion values over the efficient set. In this section, computational experiments with the Benson-Sayin heuristic algorithm are described with computational results.

For the purpose of describing these experiments, $\langle -c_1, x\rangle$ will be used to denote the objective function of problem (P). That is, $d = -c_1$ in problem (P).

To perform the computational experiments, we used the form 'max' $\{ z(x) = Cx \mid x \in X \}$ of problem MOLP, where $z(x) = (z_1(x), z_2(x), \ldots, z_p(x))$, C is the $p \times n$ criterion matrix whose rows are c_i, $i = 1,2,\ldots,p$ and $X = \{x \in \mathbb{R}^n \mid Ax \leq b,\ x \geq 0\}$ where $b \in \mathbb{R}^m$. Let $z_{max} = \max_{x \in X} \langle c_1, x\rangle$ and $z_{min} = \min_{x \in X_E} \langle c_1, x\rangle$, and let z_h be estimate of z_{min} obtained by the heuristic solution.

The computational experiments were conducted with randomly-generated problems from three categories. These three categories are defined on the basis of problem size (number of objectives × number of constraints × number of variables): 4 × 8 × 12, 5 × 12 × 16, 5 × 16 × 16. In each category, the sample size was ten. That is, in each category,

ten multiple objective linear programs were randomly generated and solved for all efficient extreme points using ADBASE (Steuer, 1983). In these ten problems, the right-hand-side elements were randomly drawn from the interval of integers [50,100]. After first providing for a 25% zero-density in the A-matrix, the remaining A-matrix elements were randomly drawn from the interval of integers [0,20]. The C-matrix elements were randomly drawn from the interval of integers [-10,20].

Each problem (P) associated with the 30 randomly generated multiple objective linear programs was next solved by the Benson-Sayin heuristic algorithm. These problems were solved on an IBM 3090 Model 600J computer, using an implementation of the heuristic algorithm written in FORTRAN. The FORTRAN program used the subroutines of OSL (IBM, 1990) to solve the required linear programming subproblems called for in the Benson-Sayin algorithm. Before using the algorithm, the user is required to supply the value for a parameter s, which decides the number of sample points in criterion space that the algorithm will generate. In addition, the user must pick the value of a parameter $M \geq 1$, which is an element of certain weighting vectors used in the algorithm [see Benson and Sayin (1991) for details]. In the computational experiments, s = 5 and M = 15 were used, based upon the problem sizes of the randomly-generated problems.

Table 4, Table 5, and Table 6 show the experiments in detail. To explain these experiments, consider the contents

of Table 4. The contents of Table 4, in which 4 × 8 × 12 means 4 objectives, 8 constraints and 12 variables are as follows. The first problem of the experiment, Problem 1, has 14 efficient extreme points. Five distinct efficient extreme points are found by using the Benson-Sayin heuristic algorithm. It is found in this problem that none of the problem's efficient extreme points has a smaller z_1 value than the value z_h found by the heuristic method. This means that the heuristic method found the minimum criterion value over the efficient set in this case. Also, the efficiency rate (%) of the heuristic solution is 100%. The efficiency rate is the percentage of the criterion value range over the efficient set that is above z_h. Mathematically, it is given by

$$Efficiency\ rate = \frac{z_{max} - z_h}{z_{max} - z_{min}}(100)$$

A similar interpretation applies to the data for the other problems in Tables 4, 5, and 6.

Table 4
$4 \times 8 \times 12$ experiment

Problem	Total eff. ext. pts	No. of eff. ext. pts. found	No. of eff. ext. pts. below z_b	Efficiency rate (%)
1	14	5	0	100.0
2	22	13	0	100.0
3	37	11	0	100.0
4	46	12	0	100.0
5	27	8	1	75.3
6	27	8	0	100.0
7	25	8	1	96.4
8	5	3	0	100.0
9	34	9	0	100.0
10	84	13	4	85.8

Table 5
$5 \times 12 \times 16$ experiment

Problem	Total eff. ext. pts.	No. of eff. ext. pts. found	No. of eff. ext. pts. below z_1	Efficiency rate (%)
1	99	17	4	87.0
2	314	14	4	99.2
3	328	19	7	96.2
4	545	18	6	87.9
5	130	11	7	76.5
6	64	13	0	100.0
7	550	23	1	99.8
8	450	20	0	100.0
9	206	14	2	89.0
10	228	20	4	91.5

Table 6
5 × 16 × 16 experiment

Problem	Total eff. ext. pts.	No. of eff. ext. pts. found	No. of eff. ext. pts. below z_h	Efficiency rate (%)
1	670	17	10	91.2
2	257	15	2	97.3
3	257	19	20	86.2
4	481	14	4	95.2
5	370	17	1	99.4
6	517	18	0	100.0
7	271	18	3	95.9
8	299	15	2	95.6
9	129	13	0	100.0
10	499	27	4	99.0

Table 7 summaries the results of the experiments. It shows (1) the average number of efficient extreme points per problem, (2) the average number of efficient extreme points found by the heuristic method, (3) the average number of efficient extreme points per problem which have z_1 values less than z_h, (4) the average percentage of the total number of efficient extreme points per problem which have z_1 values less than z_h, (5) the average efficiency rate per problem, and (6) the average central processing unit (CPU) computation time per problem.

Table 7
Average statistics for each category

	category		
Averages per problem	4 × 6 × 12	5 × 12 × 16	5 × 16 × 16
(1) No. of eff. ext. pts.	32.1	291.4	375.0
(2) No. of eff. ext. pts. found	9.0	16.9	17.3
(3) No. of eff. ext. pts. below z_h	0.6	3.5	4.6
(4) % of eff. ext. pts. below z_h	1.20	1.67	1.36
(5) Efficiency rate (%)	95.60	92.70	98.90
(6) CPU time (seconds)	0.739	1.447	1.567

Based on this computational experience, we can make some interesting observations. First, the algorithm appears quite practical even for relatively-large size problems, since there is no computational burden related to the total number of efficient extreme points. As seen in row (6) of Table 7, CPU times do not increase proportionally with the total number of efficient extreme points. Second, the Benson-Sayin heuristic solutions seem to give good estimates of minimum criterion values over the efficient set, since the efficiency rate of the heuristic solution is usually in the 66 to 100% range for

most problems in all three categories. Also, it appears from row (5) of Table 7 that the efficiency rate does not decrease as problem size increases. Third, row (4) of Table 7 shows that the average percentage of efficient extreme points that have z_i values smaller than a_i does not exceed 1.67% in all three categories. This seems to indicate that the possibility of overlooking some improved efficient solution in using the Benson-Sayin heuristic method to estimate the minimum criterion values over the efficient set is usually very small.

CHAPTER 4

USING MULTIPLE CRITERIA DECISION MAKING TO DETERMINE A BEST COMPROMISE SOLUTION FOR CITRUS ROOTSTOCK SELECTION IN FLORIDA

Since there is no single rootstock which is superior in all characteristics (see Sections 1.3 and 2.3), citrus growers generally consider using a portfolio of rootstocks in the selection process as a means of overall risk reduction. In determining the composition of their choices, growers always must consider all of the various attributes of the rootstocks simultaneously.

Section 4.1 describes the basic citrus rootstock selection problem in Florida and a general multiple objective linear programming model for this problem. Section 4.2 then describes the application of the proposed model to rootstock selection in the Fort Pierce area of Florida, which is located in St. Lucie County in southeast Florida. There are four subsections in Section 4.2. In the first, Section 4.2.1, the data used in the formulation are presented. In Section 4.2.2, with the given data, a preliminary analysis is performed. In Section 4.2.3, the problem is solved by using the interactive STEM method in two different ways. Finally, in Section 4.2.4, the results are analyzed.

4.1 Problem Statement and Model Formulation

In this section, the rootstock selection problem in Florida is stated and is formulated as a multiple criteria decision making problem.

4.1.1. Problem Statement

Before the establishment of a new grove or the replanting of a damaged grove, a grower usually selects scions which will produce the desired fruits. The selection of scions is basically based on price, yield, quality and the risks involved with the scions. After the determination of the scion, the choice of rootstocks is an important consideration, since rootstock determines horicultural characteristics of the tree (see Section 1.3).

One of the important factors affecting the decision of rootstock selection is the planting site. The ranking of the risks involved in making a decision is much different depending on the planting area. Depending on the planting area, there can be differences in many factors influencing the choice of rootstocks. Specifically, potential yields, frequency and severity of damaging freezes, drainage and availability of good quality water, and soil characteristics vary across the citrus-producing area.

Although individual circumstances are never identical, there are important considerations that may be common to most regions in Florida. These considerations include the principal performance criteria (yield and fruit quality), and the major limiting factors such as freeze, blight, and tristeza tolerance (Castle et al., 1989). Besides these major limiting factors, Phytophthora tolerance and drought tolerance are also two important factors in selecting rootstocks in Florida (McClure, 1981).

In considering the performance criteria, yield per acre over a given time frame and fruit quality should be considered. Over 85% of the citrus in Florida is processed. Therefore, fruit quality is measured by the quality of juice that can be obtained from the fruit. The major factor associated with citrus juice quality is <u>total soluble solids</u> (TSS). Citrus fruit juice solids contain a large number of soluble constituents (chiefly sugars with smaller amounts of organic acids, etc.). These are measured with a total soluble solids (TSS) hydrometer which measures specific gravity and is calibrated to measure directly the amount of total soluble solids. Total soluble solids are usually expressed as pounds per box. <u>Pounds-solids</u> are determined by measuring the TSS from a sample batch of fruit. For example, two field boxes of oranges, having 7 pounds of TSS, yield 14 pounds-solids. Growers are actually paid on a pounds-solids basis. Consequently, most citrus growers in Florida measure

their yields on the basis of TSS rather than on the basis of the number of pieces of fruit.

Freezes have had the most severe impact on tree losses in the 1980s, predominantly in Florida's northern citrus region. However, citrus blight and tristeza, have been, and continue to be, the two most important diseases causing citrus tree loss in Florida. The severity of these two diseases is highly dependent upon the citrus rootstock used in a citrus grove planting.

All rootstocks commonly used in Florida can be affected by citrus blight, but there are wide differences in susceptibility (see Section 2.3). Rough lemon has been the most seriously- affected rootstock, while Sour orange and Cleopatra mandarin have been among the least effected. Blight has not been seen in trees younger than five years, but thereafter it appears in trees of any age. Blight-affected trees may decline rapidly in some cases or slowly over a period of years. Often they remain stable for many months or even appear to recover, but subsequently degenerate into a permanent decline. Since blight-affected trees are not usually removed until they produce less than 30% of a healthy tree's yield, a citrus grower generally has more interest in the number of blight-effected trees than in the number of trees lost by blight over a certain period.

Tristeza is a viral disease that has become widespread in Florida. Various forms of tristeza affect 99.9% of all

citrus in Florida. However, Sour orange is the only rootstock commonly used in Florida that is visibly-affected by tristeza. There has been a sharp increase in tristeza in recent years and severe cases have been reported (Brlansky et el., 1986; Youtsey, 1987). Trees on most other rootstocks frequently carry the virus but are not visibly affected. As with citrus blight, affected trees may linger in an unproductive state for many years or die within a few months after symptoms first appear.

Besides these major limiting factors, the widespread occurrence of Phytophthora fungi pathogenic to citrus continues to have a significant influence on the rootstock situation in Florida. Even though much can be done to prevent serious infection by the adoption of improved cultural practices and the use of suitable preventive measures, including fungicides, it costs a great deal to perform these preventive measures on a regular basis. For this reason, citrus growers usually consider Phytophtora tolerance as one of the important factors in selecting rootstocks.

In many areas, citrus growers in Florida currently want to consider drought tolerance in selecting rootstocks, particularly because of uncertain future supplies of water. Increasing regulation in Florida may also effect the use of water for agricultural purposes in Florida.

4.1.2. The MCDM Model

The decision of rootstock selection is often a highly-individualized one and is inevitably based on compromises and assessment of the risks involved.

Generally, the major interest in short-term rootstock comparisons is in yield and fruit quality data, with disease and survival figures mentioned only in passing. There are many cases in which a citrus grower has much interest in short-term investment. In such cases, a grower usually puts the highest priority on obtaining a maximum pounds-solids yield over a short-term period. However, disease and survival are important in long-term investment. This is especially true when yield is calculated cumulatively on a per-area basis.

By noting that a performance function can be derived from the principal performances such as yield and fruit quality, the major factors which are considered as criteria in the rootstock selection process are performance, cold tolerance, blight tolerance and tristeza tolerance (see Section 4.1.1). Since there is no single rootstock which is superior in all characteristics (see Sections 1-3 and 2-3), a citrus grower is interested in achieving the best compromise combinations between his selected scion(s) and rootstocks amongst these four major factors, and in satisfying certain constraints.

The common constraints involve land availability, limitation on the maximum weighted-average susceptibility level to Phytophthora, and limitation on the maximum weighted-average damage level from drought (McClure, 1991).

Based on these considerations, the multiple criteria decision making model consists of four objective functions and three constraints. The objective functions will seek to maximize the expected performance (yield and quality) over a certain period, minimize the average cold damage level measured on some scale, minimize the number of the blight-affected trees over a certain period, and minimize the number of the visibly-affected trees due to tristeza over a certain period. The constraints will involve land availability, limitation on maximum weighted-average susceptibility level to Phytophthora measured on some scale, and limitation on maximum weighted-average damage level due to drought measured on some scale.

The number of decision variables will depend upon the number of scions which a citrus grower selects to use. For example, if a citrus grower wants to use the scions Valencia orange and Hamlin orange, the number of decision variables would be ten, since there are five commonly-used rootstocks in Florida (see Section 2.3).

The following parameters will be used in the model formulation:

m = the number of scions a citrus grower selects to use,

b_1 = the number of trees a citrus grower must plant or replant,

b_2 = a maximum desired weighted-average susceptibility level to Phytophthora measured on some scale,

b_3 = a maximum desired weighted-average damage level due to drought measured on some scale.

For simplicity of notation, let i represent indices of scion types, i = 1,2,..,m, and let j represent indices of rootstock types, j = 1,2,3,4,5 (Cleopatra mandarin, Rough lemon, Swingle citrumelo, Carrizo citrange, and Sour orange, respectively). The following notation will be used in the model formulation:

X_{ij} = the number of trees of combination i-th scion and j-th rootstock to plant (called tree of "type i-j"),

C_{1ij} = expected annual number of pounds-solids yielded per tree of type i-j over a certain period,

C_{2ij} = cold damage level of a tree of type i-j measured on some scale,

C_{3ij} = fraction of trees of type i-j affected over a certain period by blight,

C_{4ij} = fraction of trees of type i-j visibly-affected over a certain period by tristeza,

a_{1ij} = Phytophthora susceptibility level of a tree of type i-j measured on some scale,

a_{2ij} = drought damage level of a tree of type i-j measured on some scale,

$f_1(x)$ = expected annual yield in pounds-solids over a certain period,

$f_2(x)$ = average cold damage level of total tree population measured on some scale,

$f_3(x)$ = the number of trees affected over a certain period by blight,

$f_4(x)$ = the number of trees visibly-affected over a certain period by tristeza.

The scales used in measuring C_{2ij}, a_{1ij}, and a_{2ij} will be described in Section 4.2.1.

The citrus rootstock selection problem in Florida may be expressed as a multiple objective linear programming problem. This multiple objective linear program (Q) may now be stated as follows.

$$Max \quad f_1(x) = \sum_{i=1}^{m} \sum_{j=1}^{5} c_{1ij} x_{ij}$$

$$Max \quad -f_2(x) = -\frac{1}{b_1} \sum_{i=1}^{m} \sum_{j=1}^{5} c_{2ij} x_{ij}$$

$$Max \quad -f_3(x) = -\sum_{j=1}^{m} \sum_{j=1}^{5} c_{3ij} x_{ij}$$

$$Max \quad -f_4(x) = -\sum_{i=1}^{m} \sum_{j=1}^{5} c_{4ij} x_{ij}$$

subject to:

(1) $$\sum_{i=1}^{m} \sum_{j=1}^{5} x_{ij} = b_1$$

(2) $$\frac{1}{b_1} \sum_{i=1}^{m} \sum_{j=1}^{5} a_{1ij} x_{ij} \le b_2$$

(3) $$\frac{1}{b_1} \sum_{i=1}^{m} \sum_{j=1}^{5} a_{2ij} x_{ij} \le b_3$$

(4) $$x_{ij} \ge 0 \quad for \quad i = 1,2,\ldots,m \quad j = 1,2,3,4,5$$

Constraint (1) results from the fact that b_1 trees must be planted or replanted. Constraint (2) defines the limitation on the maximum desired weighted-average susceptibility level to Phytophthora measured on some scale. Constraint (3) states that a citrus grower desires to keep the maximum weighted-average drought damage level measured on some

scale below a certain level in order to avoid serious tree damage from drought.

All the coefficients used in problem (Q) should be estimated from available experimental data derived from experiments in or around the planting area. These experimental data may also sometimes be available from growers' own data based on their own experiences.

Since any solution to the rootstock selection problem is measured by using multiple objectives, the techniques of multiple criteria decision making (MCDM) provide an excellent framework for this problem. A solution to an MCDM problem is inevitably based upon the DM's relative ranking of the values involved in each objective function level, since the decision is a highly-individualized one.

With the aid of the interactive method STEM, problem (Q) allows a citrus grower to interactively explore weakly-efficient citrus rootstock selection plans. In this way, a citrus grower can explore the inherent tradeoffs available to him and eventually choose his most-preferred plan.

4.2. Model Application

The proposed model will be applied to the case of a citrus grower wishing to plant 10,000 trees in the Fort Pierce area, which is located in St. Lucie County in southeast

Florida. In this area, citrus growers generally want to use only the Valencia sweet orange scion, mainly because of high Valencia fruit quality and high dollar return (see Section 1.3). Therefore, m=1 in this application.

4.2.1. Data Collection

Since many important factors (potential yields, frequency of damaging freezes, etc.) are highly dependent on the planting area (see Section 4.1.1), the obtained data should be applicable to the Fort Pierce area, which is the area that the proposed model will be used for.

There are many difficulties in obtaining suitable data for any study about rootstocks, since research on citrus rootstocks is a time-consuming and continuing process. This is especially true for this case because no study has been designed for the rootstock selection problem. Because of this difficulty in obtaining data, some missing data were estimated by consulting with the relevant citrus rootstock researchers.

To help estimate the yield data C_{1j}, $j = 1,2,3,4,5$, required in our application of problem (Q), we consulted with Mr. J.P. McClure. Mr. McClure is a citrus grower and executive vice president of Becker Grove, Incorporated. Based upon these consultations, we obtained the data shown in Table 6.

Table 8. Twelve-Year Performance of the Valencia orange scion on selected rootstocks at Becker Grove

Rootstock	Tree Survival (%)	Yield(boxes) Cumulative Yield (per acre)	Cumulative Average Yield (per tree)	TSS/box*
Cleopatra mandarin	96	1693.4	13.62	7.4
Rough lemon	77	2502.0	18.00	6.8
Swingle** citrumelo	99	1713.1	12.32	7.4
Carrizo citrange	90	2090.6	15.04	7.5
Sour orange	99	2163.8	15.56	7.4

All trees were planted in February, 1976. Tree spacing is 12.5 by 25 feet (139 trees/acre).
*For the 1987-88 season.
**Data for Swingle citrumelo were estimated from both the performance of young trees and other sources.

We used the data in Table 8 to find estimates for the expected annual number of pounds-solids yielded per tree of each type. To accomplish this, we first annualized the numbers in Table 8 concerning the cumulative average yield per tree by dividing them by 12. For each rootstock, this yields the average annual number of boxes yielded per tree. Second,

the TSS-per-box data for the 1987-1988 year, which are shown in last column of Table 9, were used for all years in calculating average yield of pounds-solids per tree over the 1976-1988 period for each rootstock, since there are not many differences in the levels of TSS per box among the rootstocks over these years (McClure, 1991). Finally, the average annual yield of pounds-solids per tree over the 1976-1988 for Cleopatre manderin, Rough lemon, Swingle citrumelo, Carrizo citrange, and Sour orange are calculated as 8.4, 10.2, 7.6, 9.4, and 9.6 pounds, respectively. These numbers are used as the estimates for the coefficients C_{11j}, j=1,2,3,4,5, in the proposed model. Cleopatra manderin's average annual yield of pounds-solids per tree over the 1976-1988 period, which is 8.4, is calculated as follows: 8.4 = (13.62/12) × 7.4 . Since growers receive dollar returns based on pounds-solids (see Section 4.1.1), the value of the average annual yield of pounds-solids per tree is proportional to revenue.

In an attempt to estimate the coefficients C_{21j}, j=1,2,3,4,5, associated with freeze, the data from the article by R.E. Rouse, et al.(1990) were used. The December, 1989, freeze provided the opportunity to evaluate cold damage sustained by trees less than one year old in a grove at the Southwest Florida Research and Education Center at Immokalee. Twenty-seven commercial scions of citrus on most of the commercial rootstocks and on several experimental rootstocks were in a single planting site at the time of the freeze.

Damage to trees resulting from temperatures of 24° F on December 24 and 25, 1989, was evaluated in March 1990. Actual minimum temperatures recorded each morning were 24°. Two hours were recorded at or below 26° the morning of December 24 and 9 hours on December 25. Each tree was individually examined to determine the location and severity of cold damage. An interval scale, which is one of the most widely-used scale types, was used to measure the cold damage level.

In interval scale, the numbers used to rank the objects also represent equal increments of the attribute being measured. However, the location of the zero point in an interval scale is arbitrary. For example, fahrenheit and celsius temperatures are measured with different interval scales and have different zero points. For more details about scale types, see Aaker D.A. and G.S. Day (1980, Chapter 8)

The following is the interval scale used in the article by Rouse et al.(1990).

1. no damage other than to imature and succulent growth,
2. foliage loss and dieback on small terminal wood,
3. bark splits on small and medium interior wood,
4. split wood on main scaffold,
5. dead cambial tissue and split wood on the trunk.

Based on the above scale of 1-5, Rouse et al.(1990) assigned cold damage levels to rootstocks damaged in the December, 1989 freeze. The data relevant for our purposes are given in Table 9.

Table 9. Cold damage level (1-5) on selected rootstocks.

Rootstocks	Assigned number
Cleopatra mandarin	3.41
Rough lemon'	4.00
Swingle citrumelo	2.86
Carrizo citrange'	3.70
Sour orange	2.73

'From a communication with D.H. Tucker of the USDA.

The numbers given in Table 9 were used as estimates for the coefficients c_{2j}, j=1,2,3,4,5, in our application of problem (Q).

To help estimate the coefficients c_{3j}, j=1,2,3,4,5, associated with blight-affected trees, we obtained the data in Table 10 from Mr. J.P. McClure.

Table 10. Fraction of trees affected from blight over 12 years (1976-1988) at Becker Grove

Rootstocks	Fraction
Cleopatra mandarin	0.050
Rough lemon	0.270
Swingle citrumelo	0.045
Carrizo citrange	0.130
Sour orange	0.040

The numbers given in Table 10 were used as estimates for the coefficients C_{11j}, j=1,2,3,4,5, required in our application.

To find the coefficients C_{11j}, j=1,2,3,4,5, associated with tristeza, Sour orange is the only rootstock that must be considered (see Section 4.1.1). From the data given in Table 8, trees on Sour orange have relatively high yields per acre due to relatively few tree losses. However, the trees on Sour orange with other scions severely decline or die over similar periods due to the tristeza virus. By taking all scions into account, J.F. McClure estimates that 20% of the trees on Sour orange were visibly affected or dead in Becker grove over the same period of years as considered in Table 8. Since the trees on Sour orange can be visibly affected or die regardless of the scion, the fraction 1/5 can be used in estimating the coefficient of C_{111} associated with the tristeza problem for the Sour orange rootstock. The objective function value is thus interpreted as the number of trees on the Sour orange rootstock visibly affected over 12 years by tristeza.

To estimate the levels e_{11j}, j=1,2,3,4,5, of susceptibilities to Phytophthora, the data from the article by G.S. Smith, et al. (1987) was used. In this article, different rootstocks were evaluated for relative susceptibility to Phytophthora using inoculum methods. Two months after inoculation, the extent of lesion development on stems was

rated. The lesion ratings were based on a pretransformed rating scale, which is an interval scale of 0-5 given as follows:

0. Wound closed by callus formation,
1. callus formation around lesion, no vertical extension,
2. may be callus formation around,
3. no callus formation, vertical extension of lesion,
4. stem almost completely girdled, vertical extension of lesion,
5. stem completely girdled and extensive vertical extension of lesion.

The relative lesion area is calculated from the lesion development relative to an arbitrary-assigned area of stem section. The lesion area is adjusted relative to the area of the stem section to remove the effects of differential growth rates and vigor between rootstocks. Thus, the relative lesion area may be an accurate parameter to detect tolerance or an intermediate level of resistance in a rootstock (Smith et al., 1987). Table 11 shows the relative susceptibility of the rootstocks used in our application to Phythophthore based on the relative lesion area.

Table 11. Relative susceptibility levels (0-5) to Phythophthora parasitica.

Rootstocks	Relative lesion area
Cleopatra mandarin	0.600
Rough lemon	1.510
Swingle citrumelo	0.390*
Carrizo citrange	0.438
Sour orange	0.550

*From a communication with H.K. Wutcher of the USDA.

The numbers given in Table 11 were used as estimates for the coefficients a_{11j}, $j=1,2,3,4,5$, in our application.

Finally, to express the coefficients a_{21j}, $j=1,2,3,4,5$, used in the drought constraint, the data from the article by G.E. Horanic, et al.(1959) was used. In this article, the relative wilting of Valencia orange trees on the various rootstocks is reported on the basis of the following numerical rating scale, which is an interval scale of 0-3 given as follows:

0. no visible wilting;
1. a few young exterior leaves curled, slight cupping of some old leaves, fruit firm;
2. most of exterior leaves cupped and a few rolled, fruit slightly soft;
3. all exterior and most of the interior leaves curled, the remaining were flaccid, and fruit very soft but not dropping.

In September 1958 the planting of Valencia orange trees on various rootstocks in Taveres in central Florida experienced a 15-day dry period, and many trees showed severe leaf roll and softening of the fruit. Many wilted trees exhibited no overnight recovery. After a 15-day dry period, the degree of wilt for trees on various rootstocks was measured according to the above scale of 0-3. The results are given at Table 12.

Table 12 Relative wilt level of Valencia orange trees on selected rootstocks based on scale 0-3.

Rootstocks	Assigned number
Cleopatra mandarin	1.86
Rough lemon	0.80
Swingle citrumelo	1.20*
Carrizo citrange	1.00*
Sour orange	1.21

* From a communication with H.K. Wutcher of the USDA.

The numbers given in Table 12 were used as estimates for the coefficients a_{21j}, $j=1,2,3,4,5$, in our application.

To complete the specification of the data for our application, we consulted J.P. McClure, whom we chose as the decision maker for our application. Mr. McClure specified that he wanted to keep the maximum desired weighted-average susceptibility level to Phytophthora at a level between those of Carrizo citrange and Sour orange. From this consideration, he chose the number 0.5 as the maximum desired weighted-average susceptibility level to Phytophthora based on

the given scale of 0-5 (see Section 4.2.1). Mr. McClure also specified that he did not want to exceed a weighted-average wilt level of 1.4, which is in a level between those of Cleopatra mandarin and Sour orange, based on the given scale of 0-3 (see Section 4.2.1). From this information, the parameter values for b_1, b_2, and b_3 in the application are 10,000, 0.5 and 1.4, respectively.

4.2.2. Preliminary Analysis

With the data given in Section 4.2.1, the citrus rootstock selection problem for the Fort Pierce area may be solved by applying problem (Q) in an appropriate manner.

For simplicity of presentation, let z_1, z_2, z_3, and z_4 represent the objective function values for performance, average cold damage level, the number of blight-affected trees, and the number of visibly-affected trees by tristeza, respectively. Then, a decision maker (DM) must decide how to balance the goals of maximizing the expected annual yield (pounds-solids), minimizing the average cold damage level based on given scale of 1-5 (number between 1 and 5), minimizing the number of trees affected by blight (number of trees), and minimizing the number of trees visibly-affected due to tristeza (number of trees).

By noting that, for each $i \in \{2,3,4\}$, minimizing z_i is equivalent to maximizing $-z_i$, define the vector $z=(z_1, -z_2, -$

$z_3, -z_4$). Accordingly, the components of z may be expressed as the following four equations.

$$z_1 = 8.4x_{11} + 10.2x_{12} + 7.6x_{13} + 9.4x_{14} + 9.6x_{15}$$

$$-z_2 = -0.000341x_{11} - 0.00040x_{12} - 0.000286x_{13} - 0.00037x_{14} - 0.000273x_{15}$$

$$-z_3 = -0.05x_{11} - 0.27x_{12} - 0.045x_{13} - 0.13x_{14} - 0.04x_{15}$$

$$-z_4 = - 0.2x_{15}$$

Problem (Q) then becomes

Maximize z, as defined by the above four equations, subject to the following four constraints:

(1) $x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 10,000$

(2) $(1/10,000)(0.6x_{11}+1.51x_{12}+0.39x_{13}+0.438x_{14}+0.55x_{15}) \leq 0.5$

(3) $(1/10,000)(1.86x_{11} + 0.8x_{12} + 1.2x_{13} + x_{14} + 1.21x_{15}) \leq 1.4$

(4) $x_{1j} \geq 0$ for $j = 1,2,3,4,5.$

The first constraint is from the availability of land upon which 10,000 trees can be planted. The second and third constraints are from the limitation on the maximum weighted-average susceptibility level to Phytophthora measured on the given scale and the limitation on the maximum weighted-average wilt level measured on the given scale, respectively.

With the aid of ADBASE (Steuer, 1983), the following ten efficient extreme points and their criterion values are presented in Table 13 and Table 14, respectively.

Table 13 Extreme efficient points

Point	(x_{11},	x_{12},	x_{13},	x_{14},	x_{15})
A	(0,	0,	10000,	0,	0)
B	(3030,	0,	6970,	0,	0)
C	(0,	0,	0,	10000,	0)
D	(0,	0,	3125,	0,	6875)
E	(0,	982,	9017,	0,	0)
F	(4289,	0,	1558,	4153,	0)
G	(2986,	0,	4058,	2956,	0)
H	(3827,	0,	0,	6173,	0)
I	(0,	0,	0,	4464,	5536)
J	(0,	578,	0,	9422,	0)

Table 14. The criterion values

Point	(z_1,	$-z_2$,	$-z_3$,	$-z_4$)
A	(76000,	-2.860,	-450.0,	0.0)
B	(78424,	-3.026,	-485.2,	0.0)
C	(94000,	-3.700,	-1300.0,	0.0)
D	(89750,	-2.770,	-415.6,	-1375.0)
E	(78553,	-2.972,	-670.9,	0.0)
F	(86306,	-3.444,	-824.5,	0.0)
G	(84301,	-2.986,	-450.1,	-591.3)
H	(90172,	-3.589,	-993.8,	0)
I	(95107,	-3.163,	-801.8,	-1107.1)
J	(94463,	-3.717,	-1380.9,	0)

By using plan A (point A in Tables 13 and 14), the grower can expect to harvest 76,000 pounds-solids annually over a 12-year period, to obtain an average cold damage level of 2.86, to have 450 blight-affected trees over a 12-year period, and to have no visibly-affected tree by tristeza over a 12-year period. Since the grower receives from $0.80-1.75 per pound-solids depending on the season, he will expect to have anywhere from $60,800 to $133,000 annual revenue by using plan A.

From the information given at Table 14, the minimum criterion values over the efficient set for z_1, $-z_2$, $-z_3$, and $-z_4$ are 76,000, -3.717, -1380.9, and -1375.0, repectively, whereas the payoff table column minima are 89,750, -3.589, -993.8, and -1375.0. However, the Renson-Sayin heuristic method finds the exact minimum criterion values over the efficient set in this case, which would be 76,000, -3.717, -1380.9, and -1375.0, repectively. Hence, the efficiency rate (see Section 3.2) of the Renson-Sayin heuristic solution with regard to all four objective functions is 100 %, whereas the efficiency rates of the payoff table solutions with regard to the objective functions are 28%, 86%, 60%, and 100%, respectively. From these solutions, two different ranges of criterion values over the efficient set are estimated for each objective function. They can be represented as follows.

(1) Objective function 1 (Performance: Pounds-solids) :

76000 — 95107.1		(range of compromise)
76000 — 95107.1		(heuristic range)
	89750 — 95107.1	(payoff table range)

(2) Objective function 2 (Cold damage level) :

-3.717 — -2.770		(range of compromise)
-3.717 — -2.770		(heuristic range)
	-3.589 — -2.770	(payoff table range)

(3) Objective function 3 (Slight damage) :

-1380.9		-415.6	(range of compromise)
-1380.9		-415.6	(heuristic range)
	-993.8	-415.6	(payoff table range)

(4) Objective function 4 (Tristeza damage) :

-1375.0	0.0	(range of compromise)
-1375.0	0.0	(heuristic range)
-1375.0	0.0	(payoff table range)

In applying the STEM methodology to solve problem (Q) in the following section, the values obtained by using each of these two methods will be used to estimate the minimum criterion values over the efficient set.

4.2.3. Interactive (STEM) Approach

There exist a variety of solution methods for multiple objective linear programming problems (see Section 1.1.2). In practice, interactive methods have proven to be most effective in generating 'good' compromise solutions for such problems (see Steuer, 1986, p. 361). To solve problem (Q), the interactive STEM-method (Benayoun et al., 1971) was chosen

among the various procedures suited for this problem. This decision is mainly favored by the following four facts:

(1) STEM is a practical interactive procedure of proven use in exploring the weakly efficient (and hence the efficient) set of multiple objective linear programming problem (Sawaragi et al., 1985; Steuer, 1986).

(2) Since STEM can yield nonextreme final solutions rather than being restricted to only extreme points of the feasible region, the DM can potentially achieve a better solution than he can with certain other interactive procedures.

(3) Since STEM only needs the specification of aspiration levels for the objective function values, it only requires very moderate information from the DM.

(4) The computational aspects of STEM are easily implemented by solving conventional single-objective linear programming problems.

Since STEM uses the payoff method, the range of the criterion values over the efficient set may be either understated or overstated (see Section 3.1). If these ranges are incorrect, this would affect the minimax weights used in STEM, resulting in a reduction in the ability of the STEM-method to generate a variety of weakly efficient and efficient points. This problem can be ameliorated by using a better method to estimate the minimum criterion values over the efficient set. Since the Benson-Sayin heuristic method has

shown to be an effective method in estimating the minimum criterion values over the efficient set (see Section 3.2), the improvement in the efficiency of the STEM method can be easily investigated by using the solutions obtained from the Benson-Sayin heuristic procedure if this heuristic method yields better estimates of the minimum criterion values over the efficient set than the payoff table solutions.

In implementing the STEM method, the DM is allowed to relax more than one criterion value at a time, and any given criterion value more than once. This is a reasonable approach since the DM may not be too familiar with the internal structure of the problem (Q) at early stages of the process.

Before implementing the STEM method, the objective functions should be normalized in such a way that they become dimensionless quantities. There are several ways to accomplish this normalization (see Ch.8-4, Steuer, 1986). In our application, the L_1-norm is used to normalize each objective function.

To examine the improvement of the performance of the STEM method with better estimated values of the minimum criterion values over the efficient set, two approaches will be used in this dissertation. One uses payoff table solutions. The other uses the Benson-Sayin heuristic solutions.

Since the solutions generated by STEM are highly dependent upon the DM's preference structure, it is helpful to

know the DM's preference structure in order to understand the generated solution at each iteration. J.F. McClure, the decision maker in this study, specified that he had high and equal priorities on maximizing pounds-solids over a 12-year period and on minimizing tristeza damage. He had lesser priority on minimizing blight damage, and the least priority on minimizing the average cold damage level.

Mr. McClure had a high priority on maximizing pounds-solids over a 12-year period since he wanted to invest for a relatively short-term period (approximately 20 years or less). He also had a high priority on minimizing tristeza damage since he believed the forecast that the tristeza problem will be much more serious within the coming ten years than it is now. Generally, the DM's priorities are dependent on many factors, such as the planting area, the current situation, and previous experience. For example, minimizing average cold damage level would be the highest priority to a citrus grower in the central Florida area due to the frequency of freeze. Also, when grove removal for replanting is anticipated within about 12 years, maximizing pounds-solids is a very high priority.

In implementing the STEM method, payoff table solutions are first used in the subsection 4-2.3.1, and the Benson-Sayin heuristic solutions are used in the subsection 4-2.3.2. In this way, the improvement in the efficiency of the STEM-method can be easily investigated.

4.3.3.1. Using Payoff Table Solutions

To use STEM, the payoff table can be presented to the decision maker (DM) in order to demonstrate to him the ranges between the best and the worst objective function values. Based on these ranges of the objective functions, the DM selects the aspiration levels for each objective function. These levels can be adjusted at each subsequent iteration. The ranges are also used in calculating the minimax weights which are required to construct the auxiliary objective function for the single-objective linear program that is solved at each iteration of the STEM algorithm. Each weight gives the relative importance of a distance of a criterion from the ideal criterion vector (see Section 3.1). The computed weights from the payoff table corresponding to each objective function are 0.0475, 0.1916, 0.3813, and 0.3794, respectively.

Since STEM generates a member of the weakly efficient solution set in each iteration (Sawaragi et al., 1985), the DM can explore the efficient set as the iterations proceed. The ideal criterion vector can be used as a good reference point by the DM to assess the quality of the solution generated by STEM in each iteration. As long as some criterion vector components appear more satisfactory to the DM than others, he will continue iterating because the situation

can potentially be improved by making tradeoffs. To achieve tradeoffs, the DM is asked to specify which objective functions he is willing to relax and by how much he is willing to relax each one.

Table 15 and Table 16 summarize the results of the STEM iterations using this payoff table approach. Table 15 represents the generated solution at each iteration. Table 16 shows the corresponding criterion values associated with the generated solution at each iteration. It also shows which objective functions are relaxed and by how much at each iteration

In Table 16, numbers in parentheses are given for criteria that the DM is willing to relax at each iteration. The value of each such number gives the lower limit on the value of the corresponding criterion that he is willing to allow in the next iteration. For example, in the first iteration, the value of -3.40 given in parenthesis for $-z_2$ indicates that the DM will allow $-z_2$ to decrease in the next iteration, but not to a value smaller than -3.40.

Table 15. The generated solutions

Iter.	(x_{11},	x_{12},	x_{13},	x_{14},	x_{15})
1	(0,	0,	8836,	0,	64)
2	(3502,	0,	4898,	1560,	37)
3	(3853,	0,	3400,	2716,	31)
4	(3708,	0,	3532,	2259,	500)
5	(3882,	0,	2203,	3891,	25)
6	(4170,	0,	2042,	3763,	25)

Table 16. The criterion values

Iter.	Index of the relaxed criteria	Criterion values			
		(z_1,	$-z_2$,	$-z_3$,	$-z_4$)
1	2, 3	(76128,	-2.86, (-3.40)	-450.0, (-600.0)	-12.8)
2	2, 3	(81685,	-3.18, (-3.40)	-600.0, (-700.0)	-7.5)
3	2, 4	(84034,	-3.30, (-3.40)	-700.0,	-6.2) (-100.0)
4	2, 3	(84034,	-3.30, (-3.40)	-658.1, (-800.0)	-100.0)
5	2	(86158,	-3.40, (-3.50)	-800.0,	-4.97)
6		(86158,	-3.41,	-780.5,	-4.97)

In iteration 1, the solution is generated by solving the weighted minimax program, which is a single-objective linear program, using the weights computed from the payoff table solutions. With this solution, the results show that the DM wanted to relax two criterion values, the cold damage level and the number of blight-affected trees. He orally revealed that his motivation was to attempt to increase yield. In iteration 2, the criterion values of the cold damage level and the number of blight-affected trees were relaxed further. The DM told us that they were still more satisfactory to him than the criterion value for total yield. Hence, by setting the lower limit of "average cold damage level" and "the blight-affected tree" to -3.4 and -700, respectively, the DM hoped to gain an increase in annual pounds-solids. In iteration 3, the DM told us that he relaxed the criterion values of the tristeza objective function and the cold damage objective function to attempt to improve the criterion value of the performance objective function even further. The DM did not want to relax the criterion values of the blight objective function at this point, since he felt that the number of blight-affected trees, 700, was close enough to the limit of the given payoff table range, which is 993.8. In iteration 4, compared iteration 3, there is a small decrease in the number of blight-affected trees and no increase in annual pounds-solids. In iteration 4, the DM was not content with the solution, especially, he told us, because of a low

annual pounds-solids level. However, he was unwilling to sacrifice more of the criterion value for the tristeza objective function to gain more annual pounds-solids. Instead, he relaxed the criterion values of the blight objective function to -800 and of the cold damage level to -3.40 to attempt to acquire more annual pounds-solids of yield.

In iteration 5, since the DM felt that he had explored a variety of relaxations and solutions, he wanted to relax only the criterion value of the cold damage level. With the solution given at iteration 6, the DM terminated the process, because he was unwilling to further trade off any criterion value to attempt to acquire more of others.

4.2.3.2 Using the Benson-Sayin Heuristic Solutions

Since the Benson-Sayin heuristic method finds the minimum criterion values over the efficient set in this case, the ranges of the criterion values over the efficient set are neither underestimated nor overestimated. From the knowledge of these ranges, as STEM proceeds, the DM selects his aspiration levels for each objective function. Instead of payoff table ranges, these ranges are also used in calculating the weights which are required to construct the auxiliary objective function for the single-objective linear program used in STEM. The computed weights are 0.1389, 0.1752, 0.3752, and 0.3106.

Table 17 and Table 18 summarize the results of the STEM

iterations using this approach. Table 17 represents the generated solution at each iteration. Table 18 shows the corresponding criterion values associated with the generated solution at each iteration. It also shows which objective function values are relaxed and by how much at each iteration.

Table 17. The generated solutions

Iter.	(x_{11},	x_{12},	x_{13},	x_{14},	x_{15})
1	(3102,	0,	6498,	243,	157)
2	(4031,	0,	842,	5061,	66)

Table 18. The criterion values

Iter.	Index of the relaxed criteria	Criterion values			
		(z_1,	$-z_2$,	$-z_3$,	$-z_4$)
1	2, 3	(79233,	-3.05, (-3.60)	-485.3, (-900.0)	-31.5)
2		(88464,	-3.51,	-900.0,	-13.2)

In iteration 1, the solution is generated by solving the weighted minimax program, which is a single-objective linear

program, using the weights computed from the Benson-Sayin heuristic solutions. With this solution, the DM was willing to trade off two criterion values, which are the average cold damage level and the blight-affected tree numbers, to acquire more annual pounds-solids. In iteration 2, the process is terminated since the DM is satisfied with the given solution.

4.2.4 Analysis

All the solutions generated in the two different approaches are efficient solutions in this case. Also, the final solutions in the two approaches are not extreme point efficient solutions. This shows that the STEM method can converge to nonextreme final solutions rather than being restricted to only extreme points of the feasible region. In fact most solutions generated by STEM are nonextreme points, rather than extreme points, of the feasible region. This is a strength of the STEM method, especially because the DM's preference function is generally not linear. In fact, in this case, the DM preferred the final solution shown in Tables 17 and 18 to any extreme efficient solutions shown at Tables 13 and 14.

By comparing the two different results of the different approaches, the following important observations can be made.

First, STEM shows more flexibility in generating solutions by using the weights computed from the Benson-Sayin

heuristic solutions than by using the weights computed from payoff table solutions. For the performance criterion, the weight (=0.0475) computed from payoff table solutions is significantly smaller than the weight (=0.1389) computed from the Benson-Sayin heuristic solutions. Since each weight gives the relative importance of the distance to the ideal criterion vector in STEM, the payoff table weight of .0475 helps prevent STEM from generating criterion vectors which give large values of annual pounds-solids. By generating such criterion vectors, the algorithm fails to provide the DM with good solutions. This can be easily observed, for instance, by comparing the two annual pounds-solids in the solution generated in the first iteration of each approach.

Second, by computing more accurate ranges derived from the Benson-Sayin heuristic method rather than from payoff table solutions, STEM can elicit better information from the DM. This is because the DM's aspiration level for each objective function was misled by the wrong ranges computed from the payoff table solutions.

For instance, using payoff table solutions, in iteration 3 of STEM, the DM relaxed the criterion value of the tristeza objective function instead of the criterion value for the blight objective function in order to try to improve the annual pounds-solids. This is because the DM set his aspiration level for blight-affected trees to 700 or less. He felt that the number of blight-affected trees, 700, in the

solution at iteration 3 was precariously close to the worst possible value of 993.8 calculated by using payoff table solutions. However, with the actual worst possible value, 1380.9, found by using the Benson-Sayin solutions, the DM was willing to relax his aspiration level to 900 or less. By using this tradeoff information concerning the DM's aspiration level, STEM generated a more satisfactory solution to the DM which yields a larger value of annual pounds-solids.

As an another important instance, with the ranges computed from payoff table solutions, the DM never wanted to relax the criterion value of the average cold damage beyond 3.5. But with a knowledge of the range computed from the Benson-Sayin solutions, the DM relaxed his aspiration level beyond 3.5 (to 3.6). This indicates that his aspiration level for the criterion value of the average cold damage was misled by the wrong payoff table range. With better ranges computed from the Benson-Sayin heuristic method, the DM responded much better in choosing his aspiration levels for each objective. The result was that the DM found more attractive tradeoffs with the criterion values generated in the approach using the Benson-Sayin heuristic solutions.

Third, STEM needs more iterations to terminate when using payoff table solutions than when using the Benson-Sayin heuristic solutions. This is because, by presenting incorrect ranges computed from payoff table solutions and using these ranges in calculating the minimax weights, STEM is

prevented from generating certain solutions at each iteration. This causes either more iterations to find a final solution or the inability of the DM to find a final solution that pleases him.

Finally, and most important, STEM yields a final solution of better quality with the Benson-Sayin heuristic method than with the payoff table method. The DM told us that he preferred the final solution generated by using the Benson-Sayin heuristic solutions to the final solution generated by payoff table solutions. J.P. McClure, the decision maker, interpreted the two final solutions as follows. Since he was very much interested in maximizing pounds-solids, the annual 2305 pounds-solids difference between the two pounds-solids values was more important in his decision making than the differences of the other three criteria. He felt that the difference between the two criterion values for the tristeza objective function, 8.23 visibly-affected trees by tristeza over a 12-year period, was of little significance. He similarly felt that the difference between the two criterion values for the cold damage objective function, 0.1, was also insignificant, especially because his planting area has never been damaged seriously by freeze.

The DM's preference for the solution STEM yielded with the Benson-Sayin solutions can be further validated by investigating his planting trends over the past three years. He has planted 25% Cleopatra mandarin, 5% Rough lemon, 15%

Swingle citrumelo, and 55% Carrizo citrumelo among 200,000 Valencia trees over the period of 1989-91. Based on this rootstock combination with 10,000 trees, the DM would expect to harvest an average of 89,200 pounds-solids annually over a 12-year period, to achieve an average cold damage level of 3.51, to have 1042 blight-affected trees over a 12-year period, and to have no tree visibly affected by tristeza over a 12-year period. By comparing his actual planting trends with the final solutions obtained from the two different approaches, and by recalling the relatively-high importance he places on yield, the DM's preference for the final solution obtained by using the Benson-Sayin heuristic solutions over the final solution obtained by using the payoff table solutions is understandable.

Chapter 5

SUMMARY AND CONCLUSIONS

In this dissertation, a multiple objective linear programming model for the citrus rootstock selection problem has been developed. Its application to a real-world problem in the selection of Florida citrus rootstocks has been also presented. The results demonstrate that the model developed in this dissertation could be used at any citrus enterprise in Florida if the necessary data were available.

With the presented citrus selection problem, it is shown that the payoff table column minimums are significantly different from the minimum criterion values over the efficient set. By using the Benson-Sayin heuristic method, the exact minimum criterion values over the efficient set were found. Hence, in this case, the exact values of the range of compromise can be derived by the Benson-Sayin heuristic method. Two different approaches were performed in solving this problem with the interactive STEM method in order to investigate differences in performance of the STEM method. The results from these two situations were analyzed according to various criteria.

The following important conclusions can be drawn from this study:

(1) A multiple objective approach is effective in solving the citrus rootstock selection problem facing citrus growers. According to the decision maker in this case, a major benefit of the multiple objective linear programming model is its ability to illustrate the tradeoffs between the different criteria.

(2) Because an interactive STEM approach involves the decision maker during the solution process, it allows him to explore the feasible region iteratively, searching for an optimal or satisfactory, near-optimal solution. Therefore, the obtained results may be more meaningful and more likely to be utilized in the final decisions rendered.

(3) The efficiency of an interactive STEM method can be significantly improved by using a better estimate of the minimum criterion values over the efficient set.

(4) Computational experiments with the Benson-Sayin heuristic method have shown that the method can be used quite practically and efficiently with relatively-large problems in estimating the minimum criterion values over the efficient set.

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BIOGRAPHICAL SKETCH

Dongyeup Lee was born on April 17, 1957 in Seoul, Korea. He received a B.A. degree in English literature from Dongguk University in Korea in 1980. He was then drafted by the Korean Army as an officer in January of 1980 and received his discharge in June of 1982.

He attended the University of Scranton in August 1983 and obtained the degree of Master of Business Administration (MBA) in May 1985. In August of 1985 he chose to join the Ph.D. program in the Department of Decision and Information Sciences at the University of Florida. He is a member of the Operations Research Society of America.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Harold P. Benson, Chair
Professor of Decision and Information Sciences

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Selçuk S. Erenguc
Professor of Decision and Information Sciences

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Antal Majthay
Associate Professor of Decision and Information Sciences

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Charles B. Moss
Assistant Professor of Food and Resource Economics

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Chungyee Lee
Associate Professor of Industrial and Systems Engineering

This dissertation was submitted to the Graduate Faculty of the Department of Decision and Information Sciences in the College of Business Administration and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

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Dean, Graduate School